Development of LPV Models and Switching LPV-H_∞ Controller for a Hydraulic System

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***Abstract*—This paper describes the development of *linear parameter varying* (LPV) models and an LPV-H_∞ controller for one *degree of freedom* (DoF) electro-hydraulic systems. A virtual set-up consists of a hydraulic actuator driven by a servovalve and supplied from a constant-pressure power unit. The LPV models are derived based on two techniques: black-box modeling and the analytical description of the system (white box modeling). Both methods are compared to each other and the derived models have been used for the design of the switching LPV-H_∞ controller. Simulation results indicate predominance of adapting LPV framework to a hydraulic system control over conventional approaches.**

I. Introduction

The main advantages of using hydraulic actuators are related to their high power to mass ratio and possibility to separate a supply power unit from an actuator. These properties have made electro-hydraulic systems a popular choice in many branches of industry like automatic suspension systems and robotics. The aforementioned applications require new methods of precise control since the actuator cannot be considered any longer from a static point of view.

The classical approach for modeling and control of a hydraulic system is to use a simplified linear model that only captures a chosen operating point. This approach is commonly found in the industry. However, the developed controller based on the linearized model of the system does not provide acceptable performance when the operating point of the system is deviated from the point the linear model is obtained. This is why nowadays engineers are improving standard control algorithms by designing controllers that can achieve better performance and are able to tackle various nonlinear effects.

In [1] a self-regulating dynamic valve's dead-zone compensation algorithm is introduced, which gives better performance compared to a standard static compensator since the dead zone property of the valve is changing during the actuator's operation. The application of H_∞ control for an electro-hydraulic load simulator is presented in [2], where the developed controller is stable with good tracking properties.

The robust controller is also developed in [3] based on the *Quantitative Feedback Theory* to control a simple electro-hydraulic system, where uncertainties of environmental stiffness and pump pressure are considered. The problem of designing a robust controller is also examined in [4] where the robust controller is developed to deal with an uncertain valve dead-band. A sliding mode controller based on neural networks is developed in [5] to control a hydraulic system in order to eliminate the undesired chattering effect of a conventional sliding controller.

One of the best techniques that belongs to the class of nonlinear control is *linear parameter varying* (LPV) control that is an extension of the well-known gain-schedule control [6], [7]. In the gain schedule framework, sub-controllers are derived for each operating point of the system and the controller switches between these sub-controllers. The LPV control, to put it simply, can be considered as an interpolation function between these individual sub-controllers, which can guarantee stability of the system during the control operation. The idea of LPV control is well introduced in [8] and [9].

The application of LPV modeling and control to hydraulic systems are only investigated in a few works ([10], [11], [12]). The problem of robust control for an electro-hydraulic servo system driven by a double-rod actuator is studied in [10], where the dynamic model of a servovalve, a double rod cylinder and a load is considered and the H_∞ technique is used for the controller synthesis. It is shown that a parameter-dependent controller provides less overshoot than a parameter-independent controller.

The problem of a robust controller design for an electro-hydraulic system is investigated in [11]. Firstly, the hydraulic servo system is described using an LPV framework. Next, a parameter-dependent controller for reference trajectory tracking is presented. The best tracking controller is composed of a servo compensator and a stabilizing compensator.

The modeling and control of a hydraulic servo system with an LPV framework is presented in [12]. The considered set-up consists of a mass moved by a hydraulic cylinder steered by a servovalve. A new discretization algorithm is derived for LPV models and a robust synthesis framework was used to design a global H_∞ controller. Moreover, several local H_∞ controllers were designed in order to benchmark against the proposed LPV controller.

The initial simulation study of the hydraulic system considered in this work reveals its complex and nonlinear dynamical behavior, and hence, the LPV framework is considered for both modeling and control of the system. Both analytical and experimental LPV modeling techniques are tested and

This publication was made possible by NPRP grant No. 6-463-2-189 from the Qatar National Research Fund (a member of The Qatar Foundation). The statements made herein are solely the responsibility of the authors.

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978-1-5090-2188-8/16/$31.00

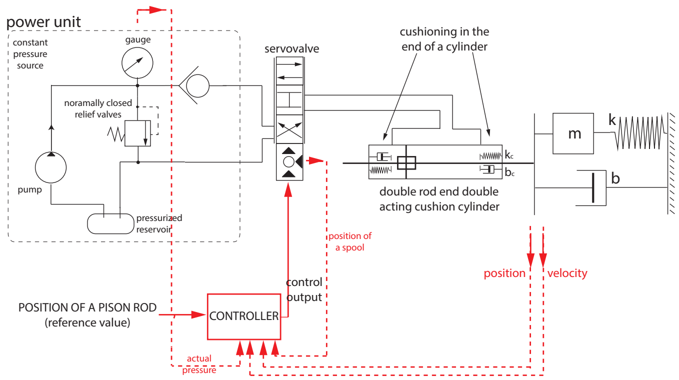

Fig. 1. Overview of the modeled system

presented in this paper. After development of LPV models, H_∞ controller is synthesized. This proposed controller allows for simultaneous optimization of robust performance and stabilization of the system. Their superiority over the classical controllers is demonstrated based on simulation results.

The aim of this paper is to show the advantages of the LPV framework for modeling and control over conventional approaches and to present a new method of controller synthesis for a hydraulic system. In relation to other existing papers about LPV control of hydraulic systems ([10], [11], [12]), this work introduces a new technique of an LPV black-box modeling, and uses a more comprehensive model of an actuator where also cushioning is considered (some kind of a shock absorber in the end of a cylinder). The rest of the paper describes the modeling techniques used (Section II), the controller design (Section III), while section IV summarizes the work.

II. Modeling

During this study a hydraulic system consisting of a linear double-acting hydraulic cylinder, a servovalve and a power unit is considered. The piston rod of the cylinder moves along a certain trajectory while interacting with a mass-spring-damper system and the position of the cylinder's piston rod is controlled by the servovalve. Fluid flow via the servovalve is controlled via an electro-magnetic coil which changes the position of a spool. The whole system is supplied by a constant pressure and it is assumed that the cylinder is cushioned. The considered hydraulic set-up is presented on the semi-hydraulic-control diagram in Figure 1.

A. Mathematical description of the hydraulic circuit

The dynamic of a load can be described as:

$$m\ddot{x} = P_L\Omega - b\dot{x} - kx, \quad x \in [-L, L], \tag{1}$$

where x is the position of the piston rod, L is the length of the cylinder, m is the mass of the load, P_L is the differential pressure between the cylinder's chambers, b is an external damping coefficient, Ω is the active area of the cylinder and k is the elasticity of the spring.

The cylinder and servovalve dynamics can be written as:

$$\frac{V_t}{4\beta_e}\dot{P}_L = -\Omega\dot{x} - C_{tm}P_L + Q, \tag{2}$$

where V_t is the total volume of the cylinder and hoses between the cylinder and the servovalve, β_e is the effective bulk modulus, C_{tm} is the coefficient of the total internal leakage, and Q is the load flow via the servovalve, which can be expressed as:

$$Q = C_d w x_v \sqrt{\frac{p_s - \mathrm{sgn}(x_v)P_L}{\rho}}, \tag{3}$$

where C_d is the discharge coefficient, ρ is the density of fluid, w is the spool valve area gradient, x_v is the spool displacement and p_s is the supply pressure.

It is assumed that the spool valve displacement x_v is related to the current input i by a first-order system:

$$\tau_v\dot{x}_v = -x_v + K_v i, \tag{4}$$

where K_v and τ_v are the servovalve's gain and time constant respectively.

The state variables are chosen as $x_1 = x$, $x_2 = \dot{x}$, $x_3 = P_L$ and $x_4 = x_v$, and the state-space representation can be derived by substituting (3) in (2) as:

$$\begin{cases} \dot{x}_1 = x_2 \\ m\dot{x}_2 = -kx_1 - bx_2 + \Omega x_3 \\ \frac{V_t}{4\beta_e}\dot{x}_3 = -\Omega x_2 - C_{tm}x_3 + \frac{C_d w}{\sqrt{\rho}}\sqrt{p_s - \mathrm{sgn}(x_4)x_3}\,x_4 \\ \dot{x}_4 = -\frac{x_4}{\tau_v} + \frac{K_v}{\tau_v}i \end{cases} . \tag{5}$$

The model also has physical constraints for the position of the piston rod and the maximum differential pressure that can be expressed as $x_1 \in (-L, L)$ and $x_3 \in (-p_s, p_s)$.

The system parameters defined in Table I are used in the numerical simulations and the system is controlled in the position mode (the aim of control is to track desired position trajectory).

B. LPV modeling

System (5) can be written in LPV framework after defining the scheduling variable θ as:

$$\theta = \sqrt{p_s - \mathrm{sgn}(x_4)x_3}. \tag{6}$$

It should be mentioned that the supply pressure p_s is not taken into account because it remains constant during the entire cylinder operation. The LPV model of the system can be written as:

$$\begin{aligned} \dot{X} &= \mathbf{A}(\theta)X + \mathbf{B}u, \\ y &= \mathbf{C}X, \end{aligned} \tag{7}$$

where $X = [x_1, x_2, x_3, x_4]^{\mathrm{T}}$, $u = i$ and the matrices $\mathbf{A}(\theta)$, $\mathbf{B}$ and $\mathbf{C}$ are given as:

$$\mathbf{A}(\theta) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & \frac{\Omega}{m} & 0 \\ 0 & -\Omega\xi & -C_{tm}\xi & \xi\kappa\theta \\ 0 & 0 & 0 & -\frac{1}{\tau_v} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_v}{\tau_v} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \tag{8}$$

TABLE I
THE PARAMETERS OF THE HYDRAULIC PLANT

symbol	description	value [unit]
m	the mass of the load	63 [kg]
k	the elasticity of the spring	10 [$\frac{\mathrm{N}}{\mathrm{m}}$]
b	the viscosity of the external damper	2 [$\frac{\mathrm{N}}{\mathrm{ms}}$]
k_c	contact elasticity	1E+5 [$\frac{\mathrm{N}}{\mathrm{m}}$]
b_c	contact damping	3E+3 [$\frac{\mathrm{N}}{\mathrm{m}}$]
$2L$	stroke of the piston rod	0.2 [m]
Ω	active area of the cylinder	7e-5 [m^2]
β_e	effective bulk modulus of fluid	2.43E+09 [Pa]
V_t	total volume of the cylinder and hoses	0.35E-05 [m^3]
C_{tm}	coefficient of total internal leakage	10 [$\frac{\mathrm{mm}^5}{\mathrm{Ns}}$]
C_d	discharge coefficient	0.7
ρ	density of fluid	1000 [$\frac{\mathrm{kg}}{\mathrm{m}^3}$]
w	spool valve area gradient	1E-04 [m^2]
p_s	supply pressure	1E+07 [Pa]
p_{max}	relief valve pressure	p_s [Pa]
τ_v	valve dynamic constant	0.001 [s]
K_v	valve gain constant	0.0005

with $\xi = \frac{4\beta_e}{V_t}$ and $\kappa = \frac{C_d w}{\sqrt{\rho}}$.

Accordingly, the scheduling variable θ in (8) changes the parameters of a linear system and makes it nonlinear. This form of LPV model is called *quasi-LPV* since θ depends on the state variables. This means that the state variable must be monitored in each computation loop and the model must adjust itself to changing conditions.

It is assumed that the cylinder is cushioned. The cushioning can have a form of a spring, pneumatic cylinder, rubber, or some other elastic material, hence it is possible to model it as a spring-damper system. In this work the case where the piston rod makes contact with the cylinder's barrel at the end of the cylinder's length is also considered. Such a situation can happen when unpredictable disturbances influence the system. In this case, the control system should still assure the stable operation, and hence the position of the piston rod is considered as another scheduling variable. This contact is modeled as a mass-spring-damper system, and in the contact case the matrix $\mathbf{A}(\theta)$ in (8) becomes:

$$\mathbf{A}(\theta) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k+k_c}{m} & -\frac{b+b_c}{m} & \frac{\Omega}{m} & 0 \\ 0 & -\Omega\xi & -C_{tm}\xi & \xi\kappa\theta \\ 0 & 0 & 0 & -\frac{1}{\tau_v} \end{bmatrix}, \tag{9}$$

while the other matrices remain the same (8).

It is assumed that contact starts when the position of the piston rod is within the following range:

$$x \in [-L, -0.9L] \vee [0.9L, L]. \tag{10}$$

The proposed hydraulic system is modeled using two methods: directly based on the analytical equations and as a black-box model.

C. White-box model

Taking into account the above mentioned parameters the white-box model can be immediate written in the affine form. For the position of the piston rod from the range $x \in (-0.9L, 0.9L)$, the model becomes:

$$\underbrace{\begin{bmatrix} A(\theta) & B \\ C & 0 \end{bmatrix}}_{S(\theta)} = \underbrace{\begin{bmatrix} A_0 & B_0 \\ C_0 & 0 \end{bmatrix}}_{S_0} + \theta \underbrace{\begin{bmatrix} A_1 & B_1 \\ C_1 & 0 \end{bmatrix}}_{S_1}, \tag{11}$$

where:

$$\begin{aligned} A_0 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & \frac{\Omega}{m} & 0 \\ 0 & -\Omega\xi & -C_{tm}\xi & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, B_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_v}{t_v} \end{bmatrix}, \\ A_1 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \kappa\xi \\ 0 & 0 & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \\ C_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, C_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{12}$$

The vertexes of the system can be found by substituting minimum (0) and maximum ($0.9p_s$) differential pressure to (6), while remembering about the positive sign of x_v when pressure is positive. By doing so the vertexes of the system become: ($\theta_{\min} = \sqrt{0.1p_s}$ and $\theta_{\max} = \sqrt{p_s}$). Hence, the model in the polytope syntax is:

$$\underbrace{\left[\begin{array}{c|c} A(\theta) & B \\ \hline C & 0 \end{array}\right]}_{S(\theta)} \in Co \left\{ \begin{array}{c} \underbrace{\left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & \frac{\Omega}{m} & 0 & 0 \\ 0 & -\Omega\xi & -C_{tm}\xi & \kappa\xi\theta_{\min} & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau_v} & \frac{K_v}{\tau_v} \\ \hline 1 & 0 & 0 & 0 & 0 \end{array}\right]}_{S_1} \\ \underbrace{\left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & \frac{\Omega}{m} & 0 & 0 \\ 0 & -\Omega\xi & -C_{tm}\xi & \kappa\xi\theta_{\max} & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau_v} & \frac{K_v}{\tau_v} \\ \hline 1 & 0 & 0 & 0 & 0 \end{array}\right]}_{S_2} \end{array} \right\} \tag{13}$$

D. Black-box model

The method used for black-box modeling can be adapted to any real-time system. The black-box identification is made in local equilibrium points by linearizing the system when changing the parameters step-by-step and then keeping them constant (so called *local method*). The black-box modeling is performed with the assumption that the relation between the scheduling variables and observable parameters is not known. Therefore, the first step is to identify what are the scheduling variables of the system. This is done by sequentially changing the input, output and by presenting constraints on the state variables in order to achieve various operating points of the system. As expected, it is observed that scheduling parameters are: differential pressure, the spool position and the position of the piston rod. However, the relation between these parameters remained unknown. Moreover, the constraints on differential pressure and position of the spool needed to be changed in such a way to be compatible with an actual hydraulic circuit. More specifically, the pressure can be negative only when the position of the valve's spool is negative and the positive

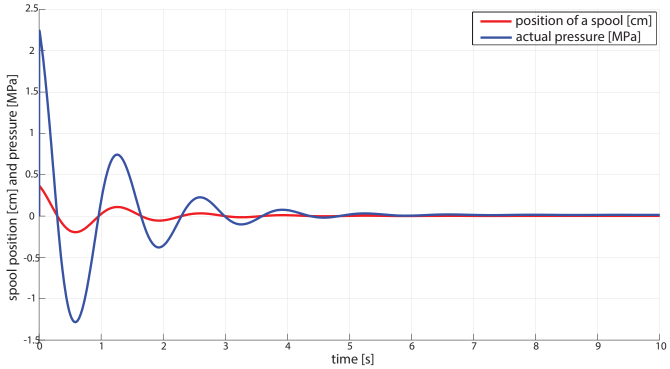

Fig. 2. Time history of position of a valve's spool and differential pressure

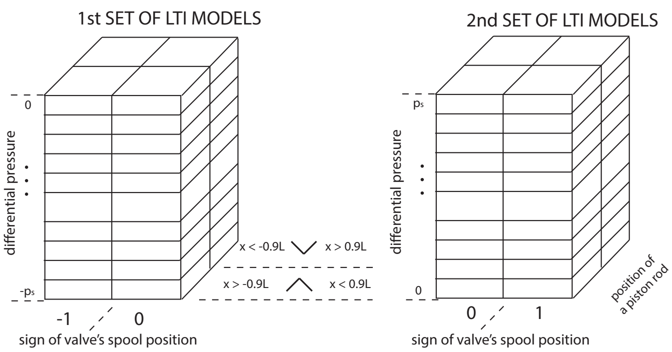

Fig. 3. Illustration of 2 sets of LTI models for the LPV black-box model

pressure corresponds to the positive position of the spool as shown in Figure 2.

It is possible that spool position and differential pressure in the actual hydraulic system have different signs. This phenomenon appears for a moment during changing direction of flow or when there is cavitation in the hydraulic circuit. However, the presented equations do not describe it since cavitation is known to be uncontrollable.

The black-box modeling leads to two separate sets of LTI models which are presented for clarification in Figure 3. These two sets comes from the fact that the negative sign of a spool corresponds to negative pressure, and vice versa. There are several techniques on how to obtain the LPV model from the set of LTI models. One of the most common, which was also adopted in this work, is to create an interpolation polynomial between the elements of the matrices. As a result, a polynomial model is obtained.

III. Controller

The complex dynamic of the system suggests that a controller which is only tuned for one operating point cannot precisely control a hydraulic system. In some hydraulic systems, this could lead to instability, and therefore, might pose a threat to people and damage expensive hydraulic equipment. The solution to this problem is to design a controller which would be capable of dealing with such multimodal dynamical systems. The LPV approach is ideal for it since the controller

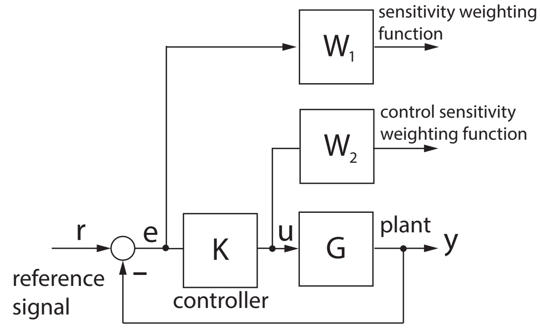

Fig. 4. Augmented plant used in the loop-shaping framework

is tuned including the properties of the system in all operating points.

In this study, an LPV H_∞ controller is used because it can tune the response over all frequencies. Therefore, it is possible to easily ensure the stability in any operating points of the system and for any type of reference signal. Simultaneous optimization of robust performance and robust stabilization is achieved based on the loop-shaping technique. This method uses shape functions in order to achieve the desired behavior of the system. As shown in Figure 4 the model of a plant needs to be augmented before performing the optimization operation. The controller is designed using a mixed sensitivity approach. According to this method the controller must meet the following criterion:

$$\left\| \begin{matrix} W_1 S \\ W_2 K S \end{matrix} \right\|_\infty < 1 , \tag{14}$$

where S is the sensitivity function of the system. The loop shaping is formulated in the frequency domain. In order to find the coefficients of the weighting functions, exhaustive iterative study is applied. The weighting sensitivity function is chosen as:

$$W_1 = \frac{0.5s + 1000}{s + 1}, \tag{15}$$

which is mainly responsible for achieving good tracking performance (it amplifies low-frequency components). The control signal weighting function is chosen as:

$$W_2 = \frac{s + 0.00025}{0.001s + 0.05}, \tag{16}$$

which penalizes the output from a controller and it is important since the valve's current cannot exceed a certain threshold. In order to reduce noise W_2 is designed to roll-off quickly beyond the desired control bandwidth.

A. H_∞ controller for the white-box model

The controller is parameter dependable and has the same order as the plant. Since the model changes its structure stepwise (it switches when the position of the piston rod crosses the assumed threshold), the controller also switches based on the current position it receives from the sensor. This comprehensive controller can be described by the following state-space equations:

$$\text{if } x > -0.9L \ \wedge \ x < 0.9L \text{ (normal operation)} :$$

$$K_1(\theta) : \begin{cases} \dot{\zeta} = \mathbf{A_{K1}}(\theta(t))\zeta + \mathbf{B_{K1}}(\theta(t))y \\ u = \mathbf{C_{K1}}(\theta(t))\zeta + \mathbf{D_{K1}}(\theta(t))y \end{cases}$$

$$\text{and if } x \leq -0.9L \ \vee \ x \geq 0.9L \text{(cylinder limit)} : \tag{17}$$

$$K_2(\theta) : \begin{cases} \dot{\zeta} = \mathbf{A_{K2}}(\theta(t))\zeta + \mathbf{B_{K2}}(\theta(t))y \\ u = \mathbf{C_{K2}}(\theta(t))\zeta + \mathbf{D_{K2}}(\theta(t))y \end{cases}$$

where A_{K1}, B_{K1}, C_{K1}, D_{K1}, A_{K2}, B_{K2}, C_{K2} and D_{K2} are state-space matrices of a controller. Each of these polytope controllers (for normal operation and cushioned area) is designed based on the white-box model, where the controller is first derived for each polytope's vertex, and then a linear combination of these sub-controllers is applied. Using the loop-shape polytope H_∞ framework, two sub-controllers are obtained for each vertex, which gives as a result 4 sub-controllers: $K_{1\alpha}$,$K_{1\beta}$,$K_{2\alpha}$,$K_{2\beta}$, where index α corresponds to a controller derived for the minimum vertex and index β corresponds to a controller derived for the maximum vertex. These sub-controllers are coupled in one controller, for instant:

$$A_{K1}(\theta) = \Lambda_{1\alpha} A_{K1\alpha} + \Lambda_{1\beta} A_{K1\beta}, \tag{18}$$

where $A_{K1\alpha}$ and $A_{K1\beta}$ correspond to state matrices of sub-controllers $K_{1\alpha}$ and $K_{1\beta}$ respectively, and the scheduling variables are described as:

$$\Lambda_{1\alpha} = 1 - \Lambda_{1\beta}, \tag{19}$$

$$\Lambda_{1\beta} = \frac{\theta - \theta_{\min}}{\theta_{\max} - \theta_{\min}}. \tag{20}$$

B. H_∞ controller for the black-box model

It is assumed while synthesizing the black-box controller that it is not known how the observable parameters depend on the scheduling parameters. Therefore, the observable state variables x_v and P_L are treated as scheduling parameters. As presented in the modeling section, this assumption leads to two separate sets of LTI models. This on the other hand, and after including the position of the piston rod as another scheduling parameter, leads to the H_∞ switching controller which consists of 4 different LPV-H_∞ sub-controllers. Switching is done based on the position of the piston rod x and based on the sign of the actual differential pressure $\text{sgn}(P_L)$ as shown in Table II. After switching between the sub-controllers, the output is calculated based on the current differential pressure (P_L) and the position of the spool (x_v). Since in this case there are two scheduling parameters, the final structure of the controller is derived based on the following equations:

$$K_{B1}(x_v, P_L) = \Lambda_\alpha K_\alpha + \Lambda_\beta K_\beta + \Lambda_\gamma K_\gamma + \Lambda_\delta K_\delta, \tag{21}$$

where

$$\begin{aligned} \Lambda_\alpha &= \frac{(x_{v\max} - x_v)}{(x_{v\max} - x_{v\min})} \frac{(P_{L\max} - P_L)}{(P_{L\max} - P_{L\min})}, \\ \Lambda_\beta &= \frac{(x_{v\max} - x_v)}{(x_{v\max} - x_{v\min})} \frac{(P_L - P_{L\min})}{(P_{L\max} - P_{L\min})}, \\ \Lambda_\gamma &= \frac{(x_v - x_{v\min})}{(x_{v\max} - x_{v\min})} \frac{(P_{L\max} - P_L)}{(P_{L\max} - P_{L\min})}, \\ \Lambda_\delta &= \frac{(x_v - x_{v\min})}{(x_{v\max} - x_{v\min})} \frac{(P_L - P_{L\min})}{(P_{L\max} - P_{L\min})}. \end{aligned} \tag{22}$$

C. Simulation Study

The two LPV controllers (one synthesized from the white-box and the other from the black-box model) and a classical PID controller are simulated and their performances are compared. The parameters of the PID controller were tuned in the one of the most common operation point ($P = 10.6$, $I = 0.06$ and $D = 12$). A semi-sinusoidal reference signal is used for validation, where the amplitude as well as the frequency of this signal change with time:

$$r(t) = t * sin(\omega(t)t). \tag{23}$$

Such a function allows to test the response of the system at various operating points. The response of the piston rod position for all controllers in shown in Figure 5. The performance of controllers is defined as an integral absolute error (IAE) in relation to the reference signal (r):

$$\epsilon = \sum_{i=1}^{n} |r_i - y_i| \tag{24}$$

where y_i denotes the i-th sample of an output signal from the plant controlled by one of the selected controllers (PID, $\text{LPV}_{\text{white}} - H_\infty$ or $\text{LPV}_{\text{black}} - H_\infty$), and n is the number of samples in the simulation. For this arbitrary chosen trajectory the results are as follows $\epsilon_{\text{white}} = 65.44\text{m}$, $\epsilon_{\text{black}} = 66.13\text{m}$ and $\epsilon_{\text{PID}} = 68.11\text{m}$, where ϵ_{white} corresponds to the IAE when using the white-box LPV controller, ϵ_{black} is the IAE when using the black-box LPV controller and ϵ_{PID} denotes the error when using the PID controller. The simulation study shows that the white-box controller is able to track the desired trajectory more accurately, while the classical PID controller achieves the worst result. The white-box controller is also better than the black-box controller. This comes from the fact that the equation of scheduling parameter is explicitly defined.

IV. CONCLUSIONS

This paper presents the LPV modeling and control for a 1-DoF hydraulic system. Two techniques of modeling are used, namely the white-box and the black-box modeling. The key issue in the black-box modeling is to impose proper constraints on the valve's spool position and on differential pressure while linearizing the model. Therefore, this approach leads to the LPV model which consists of two separate sets of scheduling variables. This also induces the structure of LPV-H_∞ controllers. This method is introduced in the literature for the first time. In the only paper which considers the LPV techniques of modeling and control of a black-box hydraulic system [12], the black-box model for varied position of a piston rod and pressure is synthesized. Therefore, the dynamic

TABLE II
THE REPRESENTATION OF THE SWITCHING BETWEEN BLACK-BOX LPV-H_∞ SUB-CONTROLLERS

	$x \leq -0.9L$ OR $x \geq 0.9L$ cylinder limit	$x > -0.9L$ OR $x < 0.9L$ normal operation
$\mathrm{sgn}(P_L) = -1$	$\dot{\zeta} = \mathbf{A_{B1}}(x_v, P_L)\zeta + \mathbf{B_{B1}}(x_v, P_L)y$ $u = \mathbf{C_{B1}}(x_v, P_L)\zeta + \mathbf{D_{B1}}(x_v, P_L)y$	$\dot{\zeta} = \mathbf{A_{B2}}(x_v, P_L)\zeta + \mathbf{B_{B2}}(x_v, P_L)y$ $u = \mathbf{C_{B2}}(x_v, P_L)\zeta + \mathbf{D_{B2}}(x_v, P_L)y$
$\mathrm{sgn}(P_L) \neq -1$	$\dot{\zeta} = \mathbf{A_{B3}}(x_v, P_L)\zeta + \mathbf{B_{B3}}(x_v, P_L)y$ $u = \mathbf{C_{B3}}(x_v, P_L)\zeta + \mathbf{D_{B3}}(x_v, P_L)y$	$\dot{\zeta} = \mathbf{A_{B4}}(x_v, P_L)\zeta + \mathbf{B_{B4}}(x_v, P_L)y$ $u = \mathbf{C_{B4}}(x_v, P_L)\zeta + \mathbf{D_{B4}}(x_v, P_L)y$

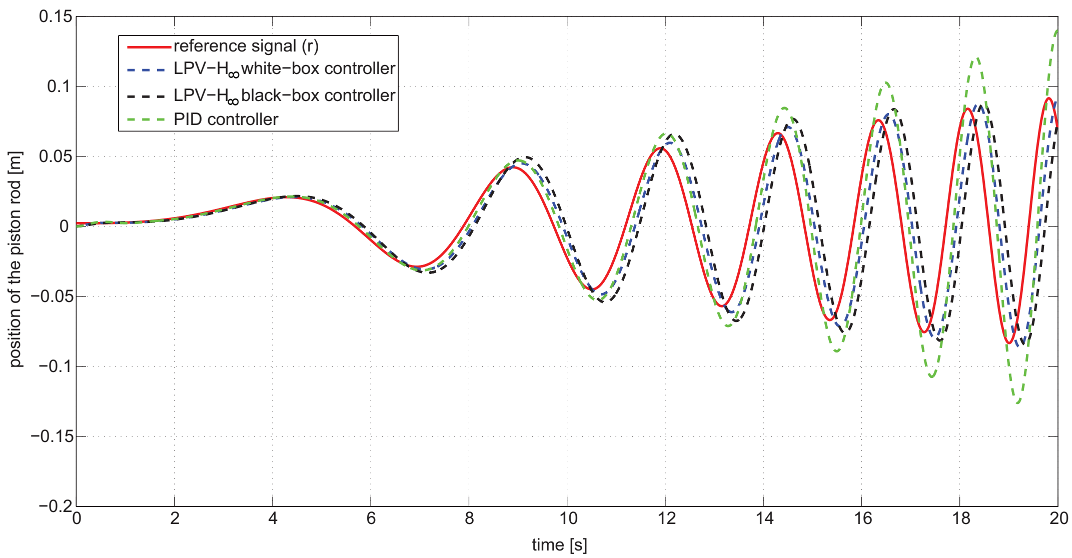

Fig. 5. The closed-loop time response of the hydraulic system (position of a piston rod) on the reference signal (*marked in red*) when using white- and black-box LPV controllers, as well as a classical PID controller

of the valve is not considered, resulting in an incomplete description of the system.

The developed controllers can be used with cushioned cylinders. This can be important in applications where the piston rod can displace to the cushioned-end region.

The black-box controller achieves inferior performance and has a more complicated structure than the controller developed based on the white-box modeling. This is expected since a good controller can be designed when the analytical model of a process is known. Nevertheless, the presented work demonstrates one possible way, which is how to design an LPV controller for a black-box hydraulic system when the position of the valve's spool and differential pressure are the scheduling variables.

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